AIAA 81-0108R

Pressure-Velocity Correlation in a Reactive Turbulent Flow

Warren C. Strahle* and Subhash B. S. Chandran†
Georgia Institute of Technology, Atlanta, Ga.

Analysis is made of the often-neglected correlation between velocity and pressure gradient fluctuations which occurs in the turbulent kinetic energy equation (TKEE). Restricting attention to this correlation for normal turbulent flames in premixed gases using high activation energy asymptotics, a model is generated for the correlation. Splitting the velocity field into solenoidal and irrotational components, it is shown that the acoustic field can dominate the vortical field under several circumstances, in which case the TKEE is not a turbulence equation but an acoustic equation. Under conditions where the vortical field is dominant the pressure-velocity correlation is shown to be essential to the TKE balance.

Nomenclature

a	= turbulence constant in Ref. 6 or width of duct
B	= pre-exponential factor
$\tilde{c}$	= Favre averaged product mass fraction in Ref. 6
c	= speed of sound
c_p	= specific heat at constant pressure
G	= Green's function
H	= heat of reaction per unit mass fuel
$\boldsymbol{i}$	= unit vector in axial direction
l	= turbulence length scale
$\tilde{L}$	= dimensionless turbulence scale of Ref. 6
M	= Mach number
p	= pressure
q	= turbulent kinetic energy
$\boldsymbol{q}$	= heat transfer vector
Q	= volumetric heat release rate
Q_∞	= $\tilde{q}_\infty/\tilde{q}_0$
$\tilde{Q}$	= dimensionless TKE of Ref. 6
r_0	= position variable of integration
$\boldsymbol{r}$	= position vector
R	= function defined by Eq. (17)
s_{mn}	= duct eigenvalues defined by Eqs. (23)
S	= function defined by Eq. (10)
S_L	= laminar flame speed
t_0	= averaging time
t	= time
T	= temperature
$\boldsymbol{u}$	= vortical component of velocity vector
u_1	= u component in axial direction
v_0	= flame speed
$\boldsymbol{v}$	= total velocity vector
V	= dimensionless flame speed in Ref. 6 or volume
$\boldsymbol{w}$	= dilatational component of velocity vector
x_i	= Cartesian coordinate in ith direction
x	= axial variable
Y_F	= mass fraction of fuel
α_1	= constant in Eq. (5)
α	= constant in pressure-velocity correlation
β	= dimensionless activation energy or turbulence parameter of Ref. 6
γ	= ratio of specific heats
δ	= preheat zone thickness
$\tilde{\delta}$	= reaction zone thickness
ϵ_1	= constant in Eq. (3)
ϵ_2	= constant in Table 1
ϵ	= TKE distribution factor of Ref. 6
$\boldsymbol{\epsilon}$	= separation distance vector
η	= constant in Eq. (3)
λ	= thermal conductivity
ξ	= dimensionless axial variable of Ref. 6
ρ	= density
τ	= T/T_0
φ	= velocity potential
Φ	= dissipation
Ψ_{mn}	= duct eigenfunctions defined by Eqs. (23)

Subscripts

0	= upstream of the flame
ω	= Fourier transform
∞	= downstream of flame
cor	= correlation
oo	= plane wave approximation

Superscripts

$(\ )^*$	= dimensional quantity
$\tilde{(\ )}$	= Favre average
$\bar{(\ )}$	= ordinary time average
$(\ )'$	= fluctuation
$(\ )''$	= Favre fluctuation

Received Nov. 17, 1980; presented as Paper 81-0108 at the AIAA 19th Aerospace Sciences Meeting, St. Louis, Mo., Jan. 12-14, 1981; revision received May 15, 1981.

*Regents Professor, School of Aerospace Engineering. Associate Fellow AIAA.

†Graduate Research Assistant, School of Aerospace Engineering.

Introduction

THERE is currently a great deal of work being performed in the modeling and computation of reactive turbulent flows. In reacting flows, such as may be found in gas turbine combustors, ramjet combustors, rocket engines, internal combustion engine cylinders, and simple laboratory turbulent flames, there is great desire for prediction capability through computation. The approaches to reactive turbulence are varied as are the degrees of success claimed by the proponents of the differing methods. One of the current avenues of approach is through higher order closure of the fluctuation equations of turbulent flows. However, when a turbulent kinetic energy equation (TKEE) is used a fundamental stumbling block occurs that has always been avoided by questionable assumptions. The problem is the occurrence of a correlation between the dot product of velocity and the pressure gradient, a correlation that usually is discarded for analytical tractability rather than for any good reason.

The pressure-velocity correlation has always given trouble in the TKEE, except in the case of incompressible isotropic turbulence where the correlation is identically zero.[1] The moment isotropy is abandoned, however, the correlation becomes important as deduced by direct[2] and indirect[3] measurement. In a relatively recent review of the incompressible problem[4] it is shown that significant progress is being made in understanding the correlation. However, the moment one moves to the compressible reacting case two important changes occur. These are 1) acoustic motions are allowed and 2) heat release tends to dominate the production of pressure. These changes will be seen to virtually negate applicability of progress in the incompressible case.

The generation of acoustic waves by heat release rate fluctuations in a turbulent flow is the problem of combustion noise,[5] and it is known that a typical pressure fluctuation level of 1% of mean pressure is accepted as normal in a combustor. For relative turbulence intensities of, say, 15%, it is a simple matter to show that for low Mach number flows products of pressure gradient and velocity are of the same order of magnitude as any other term in the TKEE. The issue is whether or not there is any correlation between the two. Moreover, there is a question of interpretation of the TKEE, which is viewed primarily as a transport equation for dominantly vortical motions. There is a tacit belief that acoustical motions probably do not belong in the TKEE because they would be ineffective in altering the turbulence, but all pressure fluctuations are not propagational in an acoustics sense, so that this belief may not be justified.

The Bray-Libby[6] theory of the turbulent flame offers a convenient starting point for investigation of pressure-velocity correlations in reactive turbulent flows. The theory, when specialized to flames normally incident to the oncoming flow, is sufficiently analytically tractable for the issue at hand and it provides an application of the TKEE. Consequently, the issues in this paper are the modeling of the pressure-velocity correlation, if it is significant, and insertion into the Bray-Libby model of the turbulent flame to deduce any interesting structural changes predicted by the model.

The exposition will require knowledge of acoustics and turbulent reacting flows. This topic is, therefore, somewhat interdisciplinary in character. Consequently, some care will be taken, beyond that normally given, to detail the developments. The procedure followed is to first of all derive two equations, replacing the conventional momentum and energy equations, which appear better suited for the issue at hand. That is, they allow more direct access to the pressure velocity correlation and the acoustics of the problem than do the more conventional forms of the conservation equations. Second, some numerical magnitudes in the model will be checked to gauge the largeness or smallness of some nondimensional groups that arise in the equations. This will enable some order of magnitude arguments to be generated as the analysis proceeds. Then it will be found that the pressure fluctuations generated in the model reacting flow are primarily acoustic in nature for high heat release flows, but that the pressure can be dominated by turbulence if one backs off somewhat in the heat release rate. This is the case of interest in this paper.

For the turbulence-driven pressure fluctuations, a model is then constructed for the pressure-velocity correlation. This model is applied to the Bray-Libby theory of a turbulent flame, and some striking structural changes are seen as compared with the original theory.

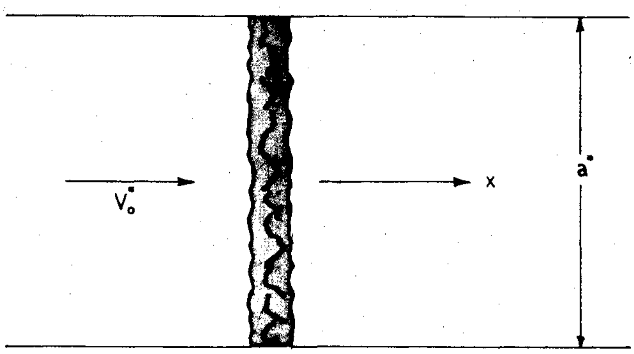

Fig. 1 Flow schematic.

Analysis

Model

In Fig. 1 it is imagined that a square duct extends from $-\infty$ to ∞ and the incoming flow is an isotropic nondecaying turbulence field at mean velocity v_0^*, which is constant across the duct. Following the arguments of Ref. 6, viscosity and, hence, turbulence dissipation in the TKEE is neglected. This allows a frozen turbulence field approaching the flame and downstream of the flame and allows a one-dimensional mean flow. This model assures the mean flow to be a potential flow and simplifies some algebra. However, a wall is inserted to properly pose the acoustics problem. That is, the walls are assumed perfectly rigid, and the infinite extent of the duct assures that only waves traveling away from the flame are allowed far upstream and far downstream of the flame. The reason the duct is square is to introduce only one characteristic dimension of the duct.

The upstream flow is considered premixed fuel and oxidizer perfect gases that can react by a global one-step irreversible reaction to form products. The activation energy will be considered high and the pre-exponential factor also high, so that, 1) the laminar flame thickness of the mixture is much smaller than the upstream integral scale of the turbulence and 2) the laminar flame would have a relatively well-defined "high activation energy structure" such as given by Bush and Fendell.[7] These assumptions, which do not violate the fast reaction Bray-Libby theory, enable a picture of the flame as a bunch of thin, locally laminar, but distorted, flamelets imbedded in the turbulent field.

Conservation Equations

All of the well-known assumptions required for the conservation equations to appear as those for a perfect gas, single component fluid with a single heat source are adopted. The length scale chosen to nondimensionalize quantities is l_0^* and the characteristic velocity is the flame speed v_0^*. Pressure is made dimensionless by p_0^*, density by ρ_0^*, and time by l_0^*/v_0^*. The equations of continuity, momentum, and energy in their more or less conventional form are

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_i}(\rho v_i) = 0 \tag{1}$$

$$\rho \frac{\mathrm{D} v_i}{\mathrm{D} t} = -\frac{\partial p}{\partial x_i} \tag{2}$$

$$\frac{\mathrm{D} p}{\mathrm{D} t} - \gamma \frac{p}{\rho} \frac{\mathrm{D} \rho}{\mathrm{D} t} = (\gamma - 1)\left(\frac{Q}{\epsilon_1} - \eta \nabla \cdot q\right) \tag{3}$$

Here effects of viscosity are omitted knowing in advance that neither the acoustics problem nor the laminar flamelet region require consideration of viscous effects.

In order to understand the nondimensionalization of the right-hand side of Eq. (3) a digression is in order based upon laminar flame theory. For a plane laminar flame Bush and Fendell[7] were the first to apply high-activation energy asymptotics to deduce a two zone structure of the flame consisting of a preheat zone where most of the temperature rise takes place followed by a thinner reaction zone. In a personal communication with Bush, however, it is uncertain to whom to ascribe the deduction of the physical size of these regions because Bush and Fendell did not work in physical dimensional space. When one goes to the high-activation energy limit the thicknesses are obvious when one looks in standard combustion texts.[8,9] Consequently, the results will be merely stated. The laminar flame speed is given by

$$S_L^* = \frac{\beta}{\rho_0^*}\left[\frac{\rho^* T^{*\alpha} \lambda^* B e^{-\beta}}{2 c_p^*}\right]^{1/2} \tag{4}$$

In Eq. (4) a Lewis number of unity has been assumed and the factor in brackets is assumed constant by an appropriate temperature dependence of the heat conductivity. In any event, the important point of Eq. (4) is the proportionality $S_L^* \propto \beta B^{1/2} e^{-\beta/2}$. Consequently, $S_L^* \to 0$ as $\beta \to \infty$ unless $B \to \infty$. The asymptotics used here will demand $\beta \to \infty$, S_L^* finite so that $B \to \infty$ is the limit process employed.

The thickness of the preheat zone is

$$\delta^* = \lambda_0^* / \rho_0^* c_{p_0}^* S_L^* \tag{5}$$

and the thickness of the reaction zone is

$$\tilde{\delta}^* = \delta^* / \beta \tag{6}$$

In a plane laminar flame the right-hand side of Eq. (3) equal to zero is the appropriate balance in the reaction zone, and in the preheat zone the last term on the left-hand side of Eq. (3) is balanced by the last term on the right. In Eq. (3) the volumetric heat release rate has been made dimensionless by its magnitude in the zone of thickness $\tilde{\delta}^*$ so that,

$$Q = Q^* / (Y_{F_0} H^* \rho_0^* S_L^* / \tilde{\delta}^*)$$

so that

$$\frac{1}{\epsilon_1} = \left(\frac{l_0^*}{\delta^*}\right) \gamma \left(\frac{Y_{F_0} H^*}{c_0^{*2}}\right) \left(\frac{S_L^*}{v_0^*}\right) \beta \tag{7}$$

is a complicated dimensionless grouping. On the other hand q^* is made dimensionless by its characteristic value in the preheat zone

$$\frac{\lambda_0^* (T_\infty^* - T_0^*)}{\delta^*} \approx \frac{\lambda_0^* Y_{F_0} H^*}{\delta^* c_{p_0}^*} = \rho_0^* S_L^* Y_{F_0} H^*$$

Therefore the parameter η is

$$\eta = \gamma \left(\frac{Y_{F_0}^* H^*}{c_0^{*2}}\right) \left(\frac{S_L^*}{v_0^*}\right) = \left(\frac{\delta^*}{l_0^*}\right) \frac{1}{\beta \epsilon_1} \tag{8}$$

In order to gain a feel for orders of magnitude involved, the quantities of Eqs. (5-8) are computed in Table 1 for numbers representative of stochiometric hydrocarbon air combustion at 1 atm pressure. Numerically, then, η has magnitude unity, ϵ_1 is a small parameter, and ϵ_2, defined in Table 1 and which is also the ratio v_0^*/v_∞^*, is also a small parameter in large heat release flows. ϵ_2 is actually contained in ϵ_1 since $Y_{F_0} H^* / c_0^{*2} = [(1/\epsilon_2) - 1]/(\gamma - 1)$. Given the magnitudes involved, since $\delta \ll 1$, it is expected that the same terms in Eq. (3) dominate in the flamelet regions as with the plane laminar flame. That is, the curvature should be large compared with the flamelet thickness. A more rigorous look at the problem of identifying laminar flamelets imbedded in turbulent regions is given Ref. 10.

Table 1 Representative values

	Assumed		
l_0^*	$=1$ cm	S_L^*	$=25$ cm/s
H^*	$=42{,}300$ J/g	v_0^*	$=30$ m/s
Y_{F_0}	$=0.067$	γ	$=1.4$
ρ_0^*	$=1.3\times10^{-3}$ g/cm^3	c_0^*	$=331$ m/s
$c_{p_0}^*$	$=1$ J/gK	λ_0^*	$=0.02$ J/K ms
E^*	$=170$ kJ/mol	T_0^*	$=300$ K
	Calculated		
$\tilde{\delta}^*$	$=0.0008$ cm	ϵ_1	$=0.0028$
δ	$=\delta^*/l_0^*=0.006$	η	$=0.3$
δ^*	$=0.006$ cm	ϵ_2	$\equiv T_0/T_\infty=0.096$
$\mathcal{R}^*$	= universal gas constant		
β	$=E^*/\mathcal{R}^*(T_\infty^*-T_0^*)=E^* c_{p_0}^*/Y_{F_0} H^* \mathcal{R}^* = 7.2$		

It is now convenient to split the velocity vector into its vortical (rotational) and dilatational (potential) components. It is common practice in acoustics to loosely identify these components as turbulent and acoustic components, respectively, although such a rigorous identification is not possible.

$$v = u + w \quad \nabla \times w = \nabla \cdot u = 0 \quad w = \nabla \varphi \tag{9}$$

It is then observed in the momentum equation, Eq. (2), after using Eqs. (9) that the introduction of a field variable S, defined by

$$\nabla S = \frac{\nabla p}{\gamma M_0^2 \rho} + \nabla \varphi_t + \nabla \frac{(\nabla\varphi \cdot \nabla\varphi)}{2} \tag{10}$$

yields

$$\nabla^2 S = -\frac{\partial}{\partial x_i}\left[\frac{\partial}{\partial x_j}(u_i u_j) + \frac{\partial}{\partial x_j}(u_j w_i) + \frac{\partial u_i}{\partial x_j} w_j\right] \tag{11}$$

Equation (11) is crucial to the following developments. It is a Poisson equation for S, which will, upon solution, yield the pressure gradient through Eq. (10). The source for S on the right-hand side of Eq. (11) is zero only if all vortical velocities are zero.

It is common practice to attack the problem of the pressure velocity correlation through construction of a Poisson equation for pressure. See, for example, Ref. 12. There are reasons that this author prefers the approach of Eq. (11), as will be evident later on. Primarily, however, density does not appear in Eq. (11).

The second crucial equation in what follows is the energy equation with Eq. (1) inserted into Eq. (3)

$$\frac{\partial p}{\partial t} + v \cdot \nabla p + \gamma p \nabla \cdot w = (\gamma - 1)\left(\frac{Q}{\epsilon_1} - \eta \nabla \cdot q\right) \tag{12}$$

The advantage of working with Eq. (12) is that it leads directly to a calculation of the potential field, under simplifications to be seen shortly.

The final conservation equation introduced is the TKEE. Here, following Ref. 6, Favre time averaging is used as well as some terms with conventional time averaging. That is, with a field variable g, in general a vector,

$$g \equiv \bar{g} + g' \qquad \bar{g} = \frac{1}{t_0}\int_0^{t_0} g(r,t)\,dt$$

$$g \equiv \tilde{g} + g'' \qquad \tilde{g} = \frac{1}{\bar{\rho} t_0}\int_0^{t_0} \rho g\,dt$$

Then the three-dimensional form of the TKEE to be used is

$$\bar{\rho}\tilde{v} \cdot \nabla \tilde{q} = -\bar{\rho}\widetilde{v_j'' v_i''}\frac{\partial \tilde{v}_i}{\partial x_j} - \frac{\partial}{\partial x_i}(\bar{\rho}\widetilde{v_i'' q''}) - \Phi - \frac{\overline{v_i''}}{\gamma M_0^2}\frac{\overline{\partial p'}}{\partial x_i} \tag{13}$$

The TKEE contains both vortical and dilatation components of the velocity and its fluctuation. The dissipation is included in Eq. (13) for completeness, but will be dropped in the following, considering the preceding remarks. A term involving the mean pressure gradient has been omitted from Eq. (13) because of the assumption of a low Mach number flow. It is the last term of Eq. (13), which is usually dropped out, that is the issue here.

In a recent work [11] it has been shown that the mean pressure gradient should not be dropped. The reason is that even though it is small, all other terms in the TKEE are small. For practical parameter values even if the Mach number is small the mean pressure gradient term contributes to the TKEE.

However, retention of this term is not central to the issues that follow, and in view of past acceptance, this term will be dropped here.

The TKEE generally is regarded as a conservation equation for q, in analogy with incompressible results where the TKE primarily is made up of the vortical velocity component. However, in the compressible reacting case it is not clear that it is a turbulence equation because the relative magnitudes of the dilatational and vortical components are not known a priori. However, one thing is clear. If the velocity vector is dominantly dilatational so that the fluctuation problem is primarily one of acoustics, the TKEE is an equation to calculate the pressure velocity correlation, not use it. The TKEE then would have nothing to do with turbulence. One of the problems that is set here is to determine under what conditions the TKEE properly may be described as a turbulence equation.

It should be noted finally that in a pure linearized acoustics problem with no mean flow the pressure-velocity correlation is identically zero. This is not true with flow and the TKEE would, as mentioned earlier, be the describing equation for the pressure-velocity correlation. So the problem appears to be one of the separation of turbulence from acoustics, which is a standard problem in aeroacoustics and one which has not been resolved satisfactorily. Only in certain limits have unambiguous results been attained and that will be found to be true here.

Some Initial Demonstrations

It is first noted that the time average of the product of a Favre fluctuation and a usual fluctuation yields the same result as the time average of the product of the two regular fluctuations. Therefore, in Eq. (13)

$$\overline{v_i''\frac{\partial p'}{\partial x_i}} = \overline{(v_i-\tilde{v}_i)\frac{\partial p'}{\partial x_i}} = \overline{(\bar{v}_i+v_i'-\tilde{v}_i)\frac{\partial p'}{\partial x_i}} = \overline{v_i'\frac{\partial p'}{\partial x_i}}$$

Consequently, for the pressure velocity correlation under investigation it does not matter whether or not the Favre or regular fluctuation in velocity is considered.

A second point to notice is that often in the following the irrotational velocity vector fluctuation will be given by a volume integral of a stochastic variable. That is, for example,

$$w' = \int_V G(r,r_0)h'(r_0)\mathrm{d}V(r_0)$$

where h' is some variable. Then if one considers $\overline{\rho' w'}$ it is seen that this is reduced tremendously in magnitude compared with, say, $(w'^2)^{1/2}$. This occurs because ρ' is only correlated with h' when the integration is passing r_0 near r. If V_{cor} is a rough measure of the volume over which ρ' and h' are correlated the following magnitudes apply:

$$(w'^2)^{1/2} \text{ is } O[\,|G|^2\,\overline{h'^2}V_{\mathrm{cor}}V]^{1/2}$$

$$\overline{\rho' w'} \text{ is } O[\,|G|\,\overline{\rho' h'}\,V_{\mathrm{cor}}] \tag{14}$$

Thus $\overline{\rho' w'}/(w'^2)^{1/2}$ is $O[(V_{\mathrm{cor}}/V)^{1/2}(\overline{\rho' h'}/(h'^2)^{1/2}]$. Even though ρ' may be $O(1/\epsilon_2)$, this ratio generally will be small because the correlation volumes to be considered will be small compared with the total volume of turbulence. The consequence is that

$$w=\bar{w}+w'=\tilde{w}+w'' \approx \tilde{w}+w'$$

and $\bar{w}\approx\tilde{w}$. That is, for the potential field, Favre and regular averages and fluctuations will be nearly identical. This is not true for the vortical field.

Potential Field

Return now to Eq. (12), introduce the mean and fluctuating quantities, and subtract off the time mean equation to yield an equation for $\nabla\cdot w'$. That is,

$$\gamma\bar{p}\nabla\cdot w' + \frac{\partial p'}{\partial t} + [v'\cdot\nabla\bar{p}+\bar{v}\cdot\nabla p' + v'\cdot\nabla p' - \overline{v'\cdot\nabla p'}$$

$$+\gamma(p'\nabla\cdot\bar{w}+p'\nabla\cdot w' - \overline{p'\nabla\cdot w'})]$$

$$=(\gamma-1)\left(\frac{Q'}{\epsilon_1} - \eta\nabla\cdot q'\right) \tag{15}$$

First, because of the low Mach number flow assumption, $\bar{p}\approx 1$, $\nabla\bar{p}\approx 0$. Second, with considerable hindsight, the term in brackets on the left-hand side may be neglected completely. This may be checked a posteriori. Third, in the turbulent flame region the first term on the left-hand side will balance the entire right-hand side with $\partial p/\partial t$ being neglected in the flame region. Fourth, outside of the flame zone the first two terms on the left will add to zero since this is the equation of acoustic motion and acoustic energy must be carried away. Outside the preheat and reaction regions of the laminar flamelet, but in the large region defined as the turbulent flame, $\nabla\cdot w'=0$. In the preheat zone $\gamma\nabla\cdot w' = -(\gamma-1)\eta\nabla\cdot q'$. In the reaction region the two terms on the right balance to within terms of $O(1/\beta\epsilon_1)$ as shown in Ref. 7 and $\gamma\nabla\cdot w'$ is $O(1/\beta\epsilon_1)$.

The neglect of $\partial p/\partial t$ in the combustion zone is known as a compactness assumption which is justified here fully. The frequencies of the noise generated will be of the same order as the frequency of the energy containing eddies,[5] $\bar{q}_0^{*\,1/2}/l_0^*$ so that the time derivative term is $O[(p'/p_0)(\bar{q}_0^{*\,1/2}/v_0^*)]$. It will be found that p'^*/p_0^* is only of the order of 1% whereas $\nabla\cdot w'$ itself is of the order of a few percent. Consequently, the $\partial p/\partial t$ term is small. Another way of saying this is that the sound source region is small compared with a wavelength. Outside the combustion region, however, the $\nabla\cdot$ operator takes on the wavelength and not l_0^*, as its characteristic length in the acoustic field. The problem is therefore one of matching a compact source field with an acoustic farfield and is a well-known problem in applied mechanics.[13]

With the preceding justification, Eq. (15) becomes in the turbulent flame zone.

$$\nabla\cdot\left(w' + \eta\frac{(\gamma-1)}{\gamma}q'\right) = \frac{(\gamma-1)}{\gamma}\frac{Q'}{\epsilon_1} \tag{16}$$

Assuming a Fourier heat conduction law with λ^* a function of T^* alone, q may be written as ∇f where f is a function of T^*. Then defining $R=\varphi+[\eta(\gamma-1)/\gamma]f$, Eq. (16) becomes

$$\nabla^2 R' = \nabla^2\left[\varphi' + \eta\frac{(\gamma-1)}{\gamma}f'\right] = \frac{(\gamma-1)}{\gamma\epsilon_1}Q' \equiv -j(r,t) \tag{17}$$

A solution is sought in terms of a Green's function satisfying

$$\nabla^2 G(r,r_0) = -\delta(r-r_0)$$

so that for a boundary enclosing the entire flame zone A, enclosing volume V

$$R'(r,t) = \int_V j(r_0,t)G\mathrm{d}V(r_0)$$

$$+\int_A(\nabla_0 j\cdot n_0 G - j\nabla_0 G\cdot n_0)\mathrm{d}A(r_0) \tag{18}$$

The appropriate condition on G is an acoustic condition for hard walls,

$$\nabla_0 G \cdot n_0 = 0$$

on the walls but a problem exists here that, in addition, j and f' must also be specified as zero. Thus, while no wall effects have been specified on the mean flow it must be demanded that some wall quenching occurs for the fluctuating flow.

It is more convenient to work with the Fourier transform of Eq. (18) whereby the condition φ_ω at $\pm\infty$ is that only outward waves can travel

$$\varphi_\omega(x \to -\infty) = C_+ e^{ik_+ x} \qquad \varphi_\omega(x \to \infty) = C_- e^{-ik_- x}$$

This condition, and all previous conditions, are satisfied for low wave number, which has been assumed, by an infinite series solution in terms of well-known eigenfunctions of the duct. The series starts with the plane wave solution and continues with exponentially decaying transverse waves, that are cut off. For order of magnitude purposes, only the plane wave term will be given here. With all details omitted, it is

$$R_{\omega_{oo}} = \int_V \mathrm{d}V(r_0) G_{oo}(x_0,x) j_\omega$$

$$G_{oo} = \frac{x}{a^2} \frac{1}{\epsilon_2^{1/2}+1} \qquad x_0 > x$$

$$= \frac{x}{a^2} \frac{\epsilon_2^{1/2}}{\epsilon_2^{1/2}+1} \qquad x_0 < x \tag{19}$$

Inverting the Fourier transforms and calculating $\nabla R'$ the result is that

$$w'_{oo} = -\frac{\eta(\gamma-1)}{\gamma} q' - i\frac{(\gamma-1)}{\gamma\epsilon_1} \int_V \frac{\mathrm{d}G_{oo}}{\mathrm{d}x} Q' \mathrm{d}V \tag{20}$$

w'_{oo} therefore consists of a local three-dimensional effect due to the heat transfer fluctuations and an axial component due to the total heat release rate fluctuations through the flame. It is noted from the magnitude of G_{oo} in Eqs. (19) that the downstream portions of the flame are most effective in producing the potential field and, from Eq. (20), w'_{oo} may locally be of order unity due to q', which is of order unity in the preheat zones. However, outside the flame zone $q'=0$. Looking at the magnitude of the plane wave velocity outside the flame zone, consider

$$\overline{w'_{oo} \cdot w'_{oo}} \approx \left(\frac{\gamma-1}{\gamma\epsilon_1}\right)^2 \overline{\left(\int_V \frac{1}{a^2} Q' \mathrm{d}V\right)^2}$$

To estimate this it is noted that Q' is correlated with Q' at neighboring points only in a volume which has thickness $\bar{\delta}$ and extending at most over the surface of the order of an eddy surface. This correlation volume is $V_{\mathrm{cor}} \approx \bar{\delta}$, in the nondimensional variables used here. Also in this volume the mean square Q' fluctuation is of order unity. However, the fraction of time that a flame is observed is small. Approximately, in an "eddy" period of order $l_0^*/\bar{q}_0^{*\,1/2}$, the time during which a flame is observed is of order $\delta^*/\bar{q}_0^{*\,1/2}$. The overall turbulent flame volume is of order a^2, assuming the flame to be only a few integral scales thick. Therefore

$$|w'| \text{ is } O\left[\frac{\gamma-1}{\gamma\epsilon_1} \frac{\bar{\delta}}{a}\right] \tag{21}$$

For the numbers of Table 1 and assuming a duct width of the order of 3 integral scales, $|w'| \approx 0.027$. For low Mach number, the pressure induced by this velocity is given by

$$|p'| = \gamma M_0 |w'| \approx 0.0034$$

The pressure is of the right magnitude, but somewhat low, when compared with common combustor experience.

It may now be verified in retrospect that the assumptions leading from Eq. (15) to Eq. (17) are valid. While neglect of the transverse waves in the w' solution probably underpredicts the magnitude of w' and p' in the flame zone, the general orders of magnitude should be preserved. The demonstration above shows that the potential velocity fluctuation can be of the order of the vortical velocity fluctuation. Consequently, the acoustic motion cannot in general be ignored in the TKEE. On the other hand, these motions are not dominant, for the numbers used here. The scaling law of Eq. (21) is therefore crucial. Rewritten

$$\frac{(\gamma-1)\bar{\delta}}{\gamma\epsilon_1 a} = \frac{(\gamma-1) Y_{F_0} H^* S_L^{*\,1/2} l_o^* (\lambda_o^*/\rho_o^* c_{po}^*)^{1/2}}{\beta^{1/2} v_o^* a^* c_o^{*\,2}}$$

Under the limit process used here, low heat release, high turbulent flame speed and high β all tend to drop this parameter. It is not desired here to include the problem of the dilatational motion. This demonstration shows what is involved in throwing out the problem of combustion noise when attempting to deal with only the vortical problem. From here on it is now assumed that the dilatation fluctuation is small compared with the vortical fluctuations.

One other problem, not addressed as yet, is the magnitude of the local fluctuation in the flame zone.

$$|w'_{\mathrm{local}}| = \frac{\gamma-1}{\gamma} \eta |\overline{q'^2}|^{1/2}$$

Forgetting for the moment the second term of Eq. (16), this is, in fact, small even though q' can be of order unity. This occurs because of the time averaging procedure. During an eddy "period" of the order of $l_0^*/\bar{q}_0^{*\,1/2}$ time units long, q' is of order 1 only during a time period of $\delta^*/\bar{q}_0^{*\,1/2}$. Consequently,

$$|w'_{\mathrm{local}}| \approx \frac{\gamma-1}{\gamma} \eta \delta^{1/2}$$

which for the numbers used here is approximately 0.007. Therefore, while locally large, this term does not contribute significantly to time averaged quantities.

Pressure-Velocity Correlation

The major outcome of the preceding demonstration is that the following magnitudes will now be assumed in a linearization of the right-hand side of Eq. (11):

$$\bar{u}=0 \qquad \bar{w} \text{ is } O(1/\epsilon_2) \qquad |w'| \ll |u'| \ll 1$$

This linearization produces

$$\nabla^2 S' = -u'_1 \frac{\mathrm{d}^2 \bar{w}}{\mathrm{d}x^2} - 2\frac{\mathrm{d}\bar{w}}{\mathrm{d}x} \frac{\partial u'_1}{\partial x} \equiv -s \tag{22}$$

Three important points are immediately apparent. These are 1) a source for S only occurs in the flame zone where $\mathrm{d}\bar{w}/\mathrm{d}x$ is nonzero, 2) only the axial component of u' is of importance as a source, and 3) the first term on the right-hand side of Eq. (11) does not appear in Eq. (22) because $\bar{u}=0$. It was precisely that term that caused pressure in a prior treatment of a pipe flow problem.[14] It would appear if the problem were one of an inclined flame with mean shear. Consequently, this problem should be revisited in future work.

The motivation for forming the equation for the S function is crucial in what follows. It would be possible, of course, to form the dot product of velocity and pressure gradient in the momentum equation and attempt a modeling of the product from the result. Equally obvious is that this would be begging the question, since the pressure-velocity correlation would be calculated from precisely the unknowns in the TKEE. In incompressible turbulence theory it is common practice[4] to obtain guidance for the pressure-velocity correlation by forming a Poisson equation for pressure, using the momentum equation; this cannot be conveniently done with Eq. (2) because density is a variable. Construction of the S function and forming a Poisson equation is the strategy here. Then, assuming certain properties of the source function, the solution of the Poisson equation brings into play properties of the duct boundaries; this adds information in addition to the right-hand side of Eq. (22). This added information is what gives credence to the result and avoids a pure question begging. In a sense, although imprecise, it is somewhat like an iterative solution. That is, first the right-hand side is assumed known and the pressure velocity correlation is calculated. Then, this first approximation is used to more precisely calculate the TKE.

A solution is attempted as in Eq. (18). However, here it is noted from Eq. (10) that $\nabla S \cdot n = 0$ is the appropriate condition on A and the boundary integral will vanish if $\nabla_0 G \cdot n_0$ is imposed on A. This will destroy any propagational plane wave for S which is actually contained in p by φ. That is, plane waves are allowed in the field variables, but by construction of the S function they disappear from the S problem. This solution is

$$S' = \sum_{m,n=0}^{\infty} 2\frac{\Psi_{mn}(y,z)}{a^2 s_{mn}} \int_V \mathrm{d}V(r_0) e^{-s_{mn}|x_0 - x|} \times \Psi_{mn}(y_0,z_0) s(r_0)$$

$$\Psi_{mn} = \cos\frac{n\pi y}{a}\cos\frac{m\pi z}{a} \qquad s_{mn} = [(n\pi)^2 + (m\pi)^2]^{1/2} \tag{23}$$

Multiplying Eq. (10) by ρ and dotting with u', together with previous assumptions, the time average of the result is

$$\frac{\overline{u' \cdot \nabla p'}}{\gamma M_0^2} = \overline{\rho u' \cdot \nabla S'} + \overline{\rho u'} \cdot i\bar{w}\frac{\mathrm{d}\bar{w}}{\mathrm{d}x} = \bar{\rho}\widetilde{u' \cdot \nabla S'} + \bar{\rho}\tilde{u}' \cdot i\bar{w}\frac{\mathrm{d}\bar{w}}{\mathrm{d}x}$$

In order to proceed to a model for this correlation the questionable assumption that Favre averages and regular time averages are equivalent will be made so that

$$\overline{u' \cdot \nabla p'} = \gamma M_0^2 \bar{\rho}\, \overline{u' \cdot \nabla S'} \tag{24}$$

will be assumed. This will be returned to in future work. For now it is noted from Eqs. (22) and (23) that a correlation of $\overline{u' u_l'}$ and $u'(\partial u_l'/\partial x)$ are involved in Eq. (24). Under Taylor's hypothesis, which should hold locally, $\partial u_l'/\partial x \approx -(1/\bar{w})\partial u_l'/\partial t$, so that if a direct correlation of $u'(\partial u_l'/\partial t)$ were involved and the turbulence were nearly isotropic, the correlation would vanish. However, the correlation actually occurs through $\partial u_l'/\partial x$ under the integral sign of Eq. (23) and thus between u' and $\partial u_l'/\partial x$ at space separated points. Nevertheless, isotropy assumed upstream and the Taylor hypothesis results in the expectation that the dominant term in the correlation comes from the first term on the right-hand side of Eq. (24). Moreover, near isotropy would imply that the dominant term of the correlation would be from the axial component of u'. Assuming this to be fact,

$$\overline{u' \cdot \nabla p'} \approx \overline{u_l' \frac{\partial p'}{\partial x}} = \gamma M_0^2 \bar{\rho} \sum_{m,n=0}^{\infty} F_{mn}$$

$$F_{mn} = \frac{2}{a^2}\Psi_{mn}(y,z)\int_V (\pm) e^{s_{mn}|x_0 - x|}\Psi_{mn}(y_0,z_0) \times \overline{u_l'(r) u_l'(r_0)}\frac{\mathrm{d}^2\bar{w}}{\mathrm{d}x^2}\mathrm{d}V(r_0)$$

where the + sign holds for $x_0 > x$ and vice versa. It is seen that the F_{mn} gradient is odd about $x = x_0$. If the velocity product correlation were symmetric about $x = x_0$ and $\mathrm{d}^2\bar{w}/\mathrm{d}x^2$ were roughly constant over the x_0 interval required for the velocity correlation to fall to zero, then a null answer would result for F_{mn}. While it appears reasonable to assign a local homogeneity assumption to $\mathrm{d}^2\bar{w}/\mathrm{d}x^2$, there must be some skew to $\overline{u_l'(r) u'(r_0)}$ because $\overline{u'^2}$ must vary through the flame. If the axial intensity derivative is positive, the positive sign in the expression for F_{mn} will be more heavily weighted than the contribution from the negative side. Then for the first few m,n terms where m/a and n/a are small and $s_{mn}|x_0 - x| \approx 0$, using the above assumptions

$$F_{mn}(\text{low } m,n) \approx \frac{2}{a^2}\Psi_{mn}^2 \frac{\mathrm{d}^2\bar{w}}{\mathrm{d}x^2}\frac{\mathrm{d}}{\mathrm{d}x}\overline{u_l'^2}$$

There are approximately a^2 terms that may be modeled like this. On the other hand, when the correlation length scale of the velocity fluctuations becomes large compared with the variation of the eigenfunction, there comes a rapid series convergence due to the oscillatory behavior of the eigenfunction. It follows that, approximately,

$$\sum_{m,n=0}^{\infty} F_{mn} \text{ is } O\left[\frac{\mathrm{d}^2\bar{w}}{\mathrm{d}x^2}\frac{\mathrm{d}}{\mathrm{d}x}\overline{u'^2}\right]$$

The following model is therefore suggested for the pressure velocity correlation when the fluctuations are dominantly vortical

$$\frac{\overline{v' \cdot \nabla p'}}{\gamma M_0^2} = \alpha\bar{\rho}\frac{\mathrm{d}\tilde{q}}{\mathrm{d}x}\frac{\mathrm{d}^2\bar{w}}{\mathrm{d}x^2} \tag{25}$$

where α is a positive constant of order unity. Whether Eq. (25) is either a source or a sink for TKE is not immediately apparent from Eq. (25); this will be investigated below. However, the major point is that Eq. (25) is apparently of the same order of magnitude as all other terms in Eq. (13). It is essential to note, because of the appearance of $\mathrm{d}^2\bar{w}/\mathrm{d}x^2$ in Eq. (25), that dilatation of the mean flow is required for the correlation to exist. Heat release is required.

Implications in the Bray-Libby Theory

Equations (25) may now be inserted into the theory of Ref. 6. One further assumption, justified by the calculations in Ref. 15, is that $\bar{w}$ may be replaced by the Favre average $\tilde{w}$. When this is done, the TKEE, Eq. (20) of Ref. 6, becomes (for flames orthogonal to the oncoming stream)

$$\frac{\mathrm{d}^2\tilde{Q}}{\mathrm{d}\xi^2} - \frac{\mathrm{d}\tilde{Q}}{\mathrm{d}\xi}\left\{1 + \frac{\alpha\tau V}{a^2\tilde{L}}\left[\frac{(1+\tau\tilde{c})}{\tilde{L}}\frac{\mathrm{d}^2\tilde{c}}{\mathrm{d}\xi^2} + \frac{\tau}{\tilde{L}}\left(\frac{\mathrm{d}\tilde{c}}{\mathrm{d}\xi}\right)^2 - \frac{\mathrm{d}\tilde{c}}{\mathrm{d}\xi}\frac{(1+\tau\tilde{c})}{\tilde{L}^2}\frac{\mathrm{d}\tilde{L}}{\mathrm{d}\xi}\right]\right\} - \frac{\epsilon\tau}{1+\tilde{c}}\left(\frac{Q_\infty}{1-Q_\infty} + \tilde{Q}\right)\frac{\mathrm{d}\tilde{c}}{\mathrm{d}\xi} = 0 \tag{26}$$

The long term in braces ({ }) multiplying $\mathrm{d}\tilde{Q}/\mathrm{d}\xi$ in Eq. (26) is the pressure velocity correlation. All other terms, representing diffusion, convection, and dilatation, were in the original equation of Ref. 6.

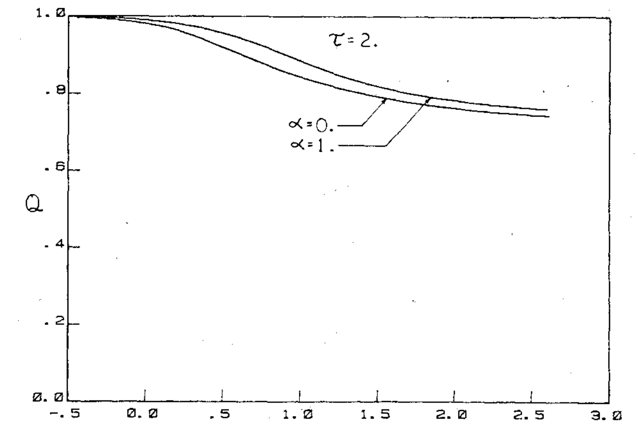

Fig. 2 Turbulent kinetic energy as a function of axial distance for a fixed heat release and various values of the pressure-velocity constant. $\tau = 2$.

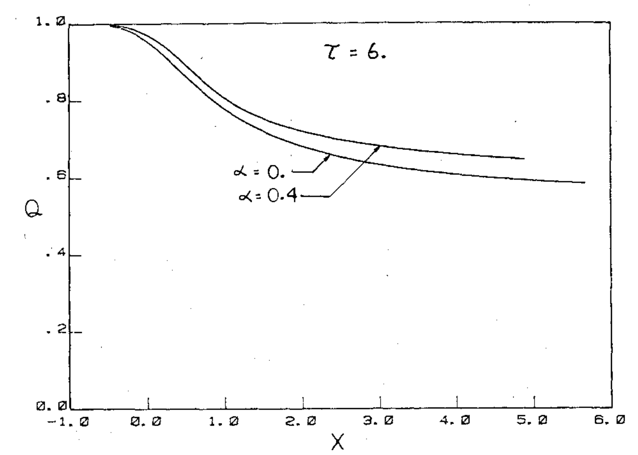

Fig. 3 Turbulent kinetic energy as a function of axial distance for a fixed heat release and various values of the pressure-velocity constant. $\tau = 6$.

There are four constants appearing in the pressure-velocity term which do not appear in the other terms. They are α, a, V, and L. In the theory of Ref. 6 the flame speed was independent of the model chosen for the length scale variation through the flame. Here that model will affect Eq. (26) and the flame speed would be dependent upon the model. In order to integrate Eq. (26) the companion equation for $\tilde{c}$ is needed from Ref. 6, but is not presented here. The integration was carried out using the same numerical technique as in Ref. 6.

Shown in Figs. 2 and 3 are the variation of $Q(\tilde{q}/\tilde{q}_0)$ as functions of nondimensional x coordinate and α. Two temperature ratios of 2 and 6 are shown. The length scale model was taken as $\tilde{L} = 1$, the type II model of Ref. 6. The values of the various parameters used in this calculation were $V = \phi/\beta = 0.4/\beta$, $a = 0.09$, and $\epsilon = 0.3$. The results show a discernible effect on the TKE, with the effect being more pronounced with higher heat release, as expected. Of course, the actual effect can be made as large as desired by varying α. The general effect appears to be a source for TKE and a consequent increase in flame speed. This is somewhat encouraging in light of the experiments discussed in Ref. 11 which generally see a rise in TKE through the flame zone.

Further work is needed on the model used here with regard to the Favre averaging procedure and to generalize the model to oblique flames. The theory should also be reworked to fit into the more advanced theory of Ref. 11. The value of α and the actual form of the correlation should be determined experimentally. Finally the appearance of $\tilde{L}$ suggests that future work will require more attention to be paid to the variation of length scale through the flame. The entire question of length scale influence on the turbulent flame speed is somewhat muddled, as discussed in Ref. 6.

Conclusions

1) In premixed reacting turbulent flows the pressure-velocity correlation is as important as any other term in the TKE balance and should not be neglected.

2) For flows with high heat release the velocity fluctuation is dilatation dominated, in which case the TKEE becomes an acoustic equation for the calculation of the correlation in question. Work is necessary on the problem of combined vortical and dilatational motions.

3) At lower heat release where vortical fluctuations dominate, a model equation has been developed for the correlation in question for flames normal to the incident stream. This model requires experimental support and leaves one constant undetermined, to be found experimentally.

4) The model predicts that, depending upon location in the flame, the pressure-velocity correlation can be either a sink or a source for TKE. However, the overall effect appears to be that of a source, tending to increase flame speed. Moreover, the model shows a dependence on turbulent length scale variation within a flame, which indicates a requirement for new modeling and experiments in this area.

5) Other than items already mentioned, further work is required on this pressure velocity correlation to a) generalize the model to oblique flames and other boundary conditions, b) remove the assumption made late in the analysis of the equivalence between ordinary and Favre averages, and c) enable its introduction into more advanced theories of the turbulent flame than used here.

Acknowledgments

This work was supported in part by the National Science Foundation under Grant CME 8022366. Dr. George K. Lea is the program monitor.

References

[1] Hinze, J. O., *Turbulence*, McGraw-Hill, New York, 1959, p. 145.

[2] Kobashi, Y., "Measurements of Pressure Fluctuation in Wake of Cylinder," *Journal of the Physical Society of Japan*, Vol. 12, May 1957, pp. 533-543.

[3] Laufer, J., "The Structure of Turbulence in Fully Developed Pipe Flow," NACA Tech Rept. 1174, 1974.

[4] Reynolds, W. C., "Computation of Turbulent Flows," *Annual Review of Fluid Mechanics*, Annual Reviews Inc., Palo Alto, 1976, pp. 183-207.

[5] Strahle, W. C., "Combustion Noise," *Progress in Energy and Combustion Science*, Vol. 4, 1978, pp. 157-176.

[6] Bray, K. N. C. and Libby, P. A., "Interaction Effects in Turbulent Premixed Flames," *Physics of Fluids*, Vol. 19, Nov. 1976, pp. 1687-1701.

[7] Bush, W. B. and Fendell, F. E., "Asymptotic Analysis of Laminar Flame Propagation for General Lewis Number," *Combustion Science and Technology*, Vol. 1, May 1970, pp. 421-428.

[8] Williams, F. A., *Combustion Theory*, Addison-Wesley, Reading, 1965, Chap. 5.

[9] Glassman, I., *Combustion*, Academic Press, New York, 1977, Chap. 4.

[10] Libby, P. A. and Williams, F. A., "Structure of Laminar Flamelets in Premixed Turbulent Flames," submitted to *Combustion and Flame*.

[11] Bray, K. N. C., Moss, J. B., and Libby, P. A., "Turbulence Production in Premixed Turbulent Flames," *Combustion Science and Technology*, Vol. 25, April 1981, pp. 127-140.

[12] Jones, W. P., "Models for Turbulent Flows with Variable Density and Combustion," *Prediction Methods for Turbulent Flows*, edited by W. Kollmann, Hemisphere, London, 1980, pp. 379-422.

[13] Pierce, A. D., *Acoustics*, McGraw-Hill, New York, 1981, pp. 183-190.

[14] Strahle, W. C. and Neale, D. H., "Turbulence Generated Pressure Fluctuations in a Rocket-Like Cavity," AIAA Paper 80-0208, Jan. 1980.

[15] Libby, P. A. and Bray, K. N. C., "Variable Density Effects in Premixed Turbulent Flames," *AIAA Journal*, Vol. 15, Aug. 1977, pp. 1186-1193.